Abstracts

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Gradings on simple Lie algebras

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A survey of some recent classification results for gradings by abelian groups on finite dimensional simple Lie algebras over algebraically closed fields will be given. This will require the classification of gradings on matrix algebras, on the algebra of octonions and on the Albert algebra (exceptional simple Jordan algebra). The tools needed from affine group schemes will be reviewed too.

Restrictions on conjugate classes

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Restrictions on conjugate classes. Relationships between upper and lower central series in groups in another algebraic structures. Classes of R. Baer and B.H. Neumann.
In this talk we desire to show the development of some topics that emerged from one of the classical results of the theory of infinite groups. We trace this expansion not only in groups, but also in other algebraic structures. We also discuss the influence of this theme on other areas, in particular, on the groups with restrictions on conjugate classes.

Asymptotic theory of finite groups

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In this paper it will be discussed some recent results and open problems in Asymptotic Theory of Finite Groups.

Groups with restrictions on infinite conjugacy classes

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A group G is called an AFC-group if for each element x of G, at least on of the indeces $|C_G(x) : \langle x \rangle|$ and $|G : C_G(x)|$ is finite. Groups with this property appear as a natural generalization of groups with finite conjugacy classes. Here some results concerning the structure of AFC-groups, and in particular the behaviour of their FC-centre, will be presented.

On some questions in noncommutative ring theory and group theory

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We will discuss some recent results on the Golod-Shafarevich algebras, nil algebras and some other ring theory topics, with applications in group theory.

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Variants of theorems of Baer and Hall on finite-by-hypercentral groups

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A classical theorem by R. Baer states that, if the m-th term $Z_m(G)$ of the upper central series a group G has finite index t in G for some positive integer m, then there is a finite normal subgroup L of G such that G/L is nilpotent of class at most m. In the opposite direction, P. Hall showed that, if there is a normal subgroup L with finite order d such that G/L is nilpotent of class at most m, then $G/Z_{2m}(G)$ has finite order bounded by a function of d and m.
Recently, in [1] it has been shown that the hypercenter of G has finite index t if and only if there is a finite normal subgroup L with order d such that G/L is hypercentral, that is coincides with its hypercenter. Then in [3] it has been shown that d may be bounded by a function of t. We have completed the picture by showing that t in turn may be bounded by a function of d.

Theorem *If a group G has a finite normal subgroup L such that G/L is hypercentral (resp. nilpotent of class m), then the hypercenter of G (resp. $Z_{2m}(G)$) has index bounded by a function of $|L|$.*

As an application we generalize results from [2].

Theorem *Let A be a finite-by-hypercentral group of automorphisms of a group G such that $A^{Inn(G)} = A$. If there is an ascending normal series in G with a finite number of finite factors and such that A acts trivially on all other factors, then:*
i) there is a finite index normal A-subgroup G_0 of G such that A stabilizes an ascending G-series of G_0;
ii) there is is a finite normal A-subgroup L such that A stabilizes an ascending G-series of G/L.

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Additivity of topological entropy for locally profinite groups

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We consider the topological entropy h_{top} for continuous endomorphisms $\phi : G \to G$ of totally disconnected locally compact groups G. We show the additivity of h_{top}, that is,

$$h_{top}(\phi) = h_{top}(\phi \restriction_H) + h_{top}(\bar{\phi}) \, ,$$

where H is a closed ϕ-stable subgroup of G that is either normal or compact and contains $\ker(\phi)$, and $\bar{\phi} : G/H \to G/H$ is the map induced by ϕ. As an application we show the precise relation of the topological entropy $h_{top}(\phi)$ with the scale $s(\phi)$ introduced by Willis.

F-norm and the induced automorphisms in finite groups

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The norm $N(G)$ of a finite group G is the intersection of the normalizers of all subgroups of G, which was first introduced by Baer in 1934. Recently years, it is a quite interesting to investigate some kind of "generalized norms". The $\mathcal{F}$-norm $N_{\mathcal{F}}(G)$ of a finite group G is the intersection of the normalizers of the $\mathcal{F}$-residual of all subgroups in G, where $\mathcal{F}$ is a non-empty formation and $H^{\mathcal{F}}$ is the $\mathcal{F}$-residual of H. In this talk we introduce our recent work about $\mathcal{F}$-norms and the induced automorphisms in finite groups.

Groups in which every finite subnormal subgroup is normal

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A group G is called a T-group if normality in G is a transitive relation, i.e. if all subnormal subgroups of G are normal. The structure of soluble T-groups is well-known, and several authors have investigated groups in which the normality condition is imposed only to a relevant system of subnormal subgroups. We consider here (generalized) soluble groups in which all finite subnormal subgroups are normal.

Stallings' decomposition theorem for totally disconnected locally compact groups

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It will be shown an analogue of Stallings' decomposition theoremfot totally disconnected locally compact (= t.d.l.c.) groups, i.e., a t.d.l.c. group G with more then one end splits either as an HNN-extension $H*_K^t$ where K is a compact open subgroup, or non-trivially as a free product with amalgamation $H *_K J$, where J and K are compact and open. This splitting theorem can be formulated in terms of rational discrete cohomology of G with coefficients in the standard bimodule $Bi(G)$. Moreover, using an accessability results due to Y. Cornulier, an analogue of Karrass-Pietrowski-Solitar theorem for t.d.l.c groups will be proved, i.e., a compactly presented t.d.l.c. group G of rational discrete cohomological dimension at most 1 is isomorphic to the fundamental group $\Pi_1(\mathcal{G}, \Lambda)$ of a finite graph of profinite groups $(\mathcal{G}, \Lambda)$.

Algebras with polynomial growth of their codimensions

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Let A be an associative algebra over a field F of characteristic zero and let $c_n(A), n = 1, 2, \dots,$ be its sequence of codimensions.
It is well known that if A satisfies some non-trivial polynomial identity then the sequence of codimensions of A is exponentially bounded. Moreover either $c_n(A)$, $n = 1, 2, \dots,$ is polynomially bounded or $c_n(A)$ grows exponentially.
The purpose of this note is to present some results about algebras whose codimensions are polynomially bounded.

Groups of Infinite Rank with Normality Conditions on Subgroups

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A group G is said to have finite (Prüfer) rank r if every fnitely generated subgroup of G can be generated by at most r elements and r is the least positive integer with such property. If such an r does not exist, G is said to have infinite rank. In this talk, (generalized) soluble groups in which the subgroups of infinite rank satisfy some normality conditions are considered.

Profinite groups with a cyclotomic p-orientation

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For a profinite group G a p-orientation for G is a continuous group homomorphism $\theta\colon G \to \mathbb{Z}_p^\times$, with $\mathbb{Z}_p^\times$ the group of units of the ring of p-adic integers $\mathbb{Z}_p$. If the induced twist $\mathbb{Z}_p[\![G]\!]$-module $\mathbb{Z}_p(1)$ (i.e., $g.\lambda = \theta(g) \cdot \lambda$) satisfies certain conditions – in particular, the continuous cohomology group $H^2(G, \mathbb{Z}_p(1))$ is torsion-free –, then θ is said to be cyclotomic. Cyclotomic p-orientations arise naturally from some Poincaré duality groups (e.g., Demushkin groups and powerful pro-p groups) and from absolute Galois groups. Then, we can say much about the structure of pro-p groups equipped with a cyclotomic p-orientation: for example, an Artin-Schreier type result holds for these groups; moreover, such a pro-p group G has a distinguished subgroup, the θ-centre Z_θ, which is the maximal abelian normal subgroup of G, and we may decompose G via a split central extension $1 \to Z_\theta \simeq \mathbb{Z}_p(1)^n \to G \to G_\circ \to 1$ (and the non-triviality of Z_θ has very relevant consequences in the arithmetic case). This is a joint work with Thomas Weigel.

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Uncountable Groups with Restrictions on Subgroups of Large Cardinality

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In the last few decades many results, showing the strong influence of proper large subgroups, were brought to light. Anyway most of these deal with the concept of "rank"; a group G is said to have *finite Prüfer rank* r if every finitely generated subgroup of G can be generated by at most r elements, and r is the least positive integer with such a property.
Here we attach the general problem from a different (and maybe more natural) point of view: studying the influence of subgroups which have large cardinality.

On large cycle conditions in finite groups

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For a finite group G, let $\lambda(G)$ denote the largest fraction of elements of G lying on a single cycle of some automorphism of G. For a fixed $\rho \in (0,1)$, we say that G satisfies the large cycle condition with respect to ρ if and only if $\lambda(G) > \rho$. In this talk, we explore two examples of how such large cycle conditions restrict the structure of G. More precisely, we present sketches of proofs of our recent results that $\lambda(G) > 1/2$ implies that G is abelian and that $\lambda(G) > 1/10$ implies that G is solvable (where both bounds are best possible). We also point out an interesting crosslink to pseudorandom number generation.

Orthogonal bases of Brauer symmetry classes of tensors for groups with cyclic support on non-linear Brauer characters

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In this talk, I will provide a formula for finding dimension of Brauer symmetry classes of tensors associated with the irreducible Brauer characters of the groups with cyclic groups support on non-linear Brauer characters. I also will discuss the necessary and sufficient condition for the existence of the o-basis of Dicyclic groups, Dihedral groups and Semi-dihedral groups.

Submonoids of nilpotent groups and their algebras

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Group algebras of finitely generated nilpotent groups (and, more generally, of polycyclic-by-finite groups), have received a lot of attention. They provide a rich family of examples

of noetherian algebras, and have a number of intriguing properties. In particular, these include properties of the prime spectrum, of the division quotient rings (in case of torsion free groups), and certain arithmetical properties.
Algebras of submonoids of nilpotent groups have not been extensively studied beyond the special case of noetherian algebras. The first aim of the talk is to present a recent example that motivated a lot of interest in algebras of this type. This motivating example comes from some aspects of noncommutative geometry. We will also recall some of the most spectacular results on group algebras of finitely generated nilpotent groups. Finally, recent results on submonoids of nilpotent groups and on their algebras will be presented. Some of the open problems in this area, that rely on questions concerning rewriting elements in finitely generated nilpotent groups, will be stated. The presented new results come from a joint work with Eric Jespers.

The influence of arrangement of subgroups on the group structure

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Investigation of groups satisfying certain related to arrangement of subgroups conditions allows algebraists to introduce and describe many important classes of groups. Most of these conditions are based on the fundamental notion of normality and built with the help of this concept different subgroup chains (series). Some of important results obtained on this way we will discuss in the talk.

A group theoretical version of Hilbert's theorem 90

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It is somehow astonishing that the classical Hilbert 90 property holds also in a group theoretical context, i.e., one has the following.

Theorem 1 (C. Quadrelli & T. W.). *Let N be a co-cyclic normal subgroup of the group G, i.e., there exists $x \in G$ such that $G/N = \langle xN \rangle$. Then*

$$[G, G] = [x, N] \cdot [N, N],$$

where $[x, N] = \{ [x, n] \mid n \in N \}$. In particular, one has

$$kernel(i_{N,G}) = (x - 1) \cdot N^{\mathrm{ab}},$$

where $i_{N,G} \colon N^{\mathrm{ab}} \to G^{\mathrm{ab}}$ is the canonical map and $G^{\mathrm{ab}} = G/[G, G]$.

Although one can prove this fact by very elementary means it has some remarkable consequences. The first to be mentioned is the following.

Theorem 2 (C. Quadrelli & T.W.). *Let G be a finitely generated pro-p group and N an open co-cyclic normal subgroup such that for all H, $N \subseteq H \subseteq G$, H^{ab} is torsion free. Then N^{ab} is a $\mathbb{Z}_p[G/N]$-permutation lattice.*

The group theoretical version of Hilbert's theorem 90 can also be applied to some classical problems in number theory. E.g., it induces the following quantitative version of Hilbert's theorem 94.

Theorem 3 (C. Quadrelli & T.W.). *Let L/K be a finite cyclic Galois extension of number fields, and let S be a finite set of places containing all infinite places such that L/K is unramified outside S and completely split for all places lying in S. Then*

$$|\mathbf{capk}(L/K, S)| = |L : K| \cdot |\mathbf{capc}(L/K, S)|,$$

where $\mathbf{capk}(L/K, S) = \ker(\mathcal{O}_K(S) \to \mathcal{O}_L(S))$ *denotes the* **capitulation kernel** *of the extension, and* $\mathbf{capc}(L/K, S) = \operatorname{coker}(\mathcal{O}_K(S) \to \mathcal{O}_L(S)^{\mathrm{Gal}(L/K))})$ *denotes the* **capitulation cokernel** *of the extension.*

The objective of the talk will be to show how these results can be deduced from the group theoretical version of Hilbert's theorem 90. If time permits we will also present a general **Schreier formula** for cocyclic normal subgroups of prime index.
This is a joint work with Claudio Quadrelli.

Some contributions to the saturated fusion systems

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In this talk, we will introduce the concept of *permutable length* for a nilpotent finite group is less or equal to its nilpotent class. The main result we would like to present in this talk is that the p-length of a p-soluble finite group is upper-bounded by the permutable length of its Sylow p-subgroups. This result has improved the celebrated Hall-Higman p-length theorem, which stated tha the p-length of a p-soluble finite group is upperbounded by the nilpotent class of its Sylow p-subgroups.

Generators for discrete subgroups of 2-by-2 matrices over rational quaternion algebras

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In [1], we developed an algorithm to determine generators for discrete subgroups of 2-by-2 matrices over quadratic extensions of $\mathbb{Q}$. These groups act discontinuously on hyperbolic 3-space and the algorithm constructs a fundamental domain to find a set of generators. In this work we try to imite this algorithm to Clifford matrices acting discontinuously on hyperbolic 5-space in order to get a set of generators for discrete subgroups of 2-by-2 matrices aver quaternion algebras.

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Groups with a restriction on normalizers or centralizers

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The analysis of groups which satisfy some restriction related to normality is a common topic in Group Theory. Recently, in [3] and [4], a new condition has been considered in the realm of finite nilpotent groups: if k is a fixed positive integer, what can be said about a finite p-group G which satisfies $|N_G(H) : H| \leq p^k$ for every non-normal subgroup H of G, and which is not a Dedekind group? A related problem can be raised about centralizers of elements. Thus one may ask what can be said about G if $|C_G(x) : \langle x\rangle| \leq p^k$ for every non-normal cyclic subgroup $\langle x\rangle$ of G.

In this talk we present some new results on finite p-groups with this kind of restrictions [1], and we focus on infinite groups with the corresponding finiteness conditions on normalizers of subgroups and centralizers of elements [2].

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Groups Factorized by Mutually Permutable Subgroups

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A group $G = AB$ is said to be mutually factorized by its subgroups A and B if $AY = Y$ A and $BX = XB$ for every subgroup X of A and Y of B.
Starting from a result proved by J. C. Beidleman and H. Heineken in the case of finite groups, the embedding properties of the commutator subgroups A' and B' in the infinite group G, mutually factorized by its subgroups A and B, are studied, under suitable finiteness conditions on the rank.

Some results on Golod-Shafarevich and quadratic algebras and semigroups

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We give an overview of our results on quadratic algebras, especially on the circle of questions related to the famous Golod-Shafarevich estimate on the Hilbert series of algebras presented by generators and relations. Those and related results could be found in the following papers:

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The Hochschild product of radical braces

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Catino and Rizzo [*Regular subgroups of tha affine group and radical circle algebras*, Bull. Aust. Math. Soc. 79 (2009), 103-107] established a link between regular subgroups of the affine group and the radical brace over a field on the underlying vector space. We propose new constructions of a radical braces that allow us to obtain systematic contructions of regular subgroups of the affine group. In particular, this approach puts in a more general context the regular subgroups contructed by Tamburini Bellani in [*Some remarks on regular subgroups of the affine group*, Int, J. Group Theory, 1 (2012), 17-23].

The asymmetric product of radical braces

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In this talk we are going to introduce a new radical brace construction which extended the semidirect product of two radical braces. In the case of braces over a field, we may obtain some regular subgroups of the affine group and this approach allows us to generalize the regular subgroups constructed by Hegedüs in *Regular Subgroups of the Affine Groups, J. Algebra* ***225*** *(2000), 740–742.*

On some arithmetic properties of finite groups

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Let $\sigma = \{\sigma_i | i \in I\}$ be some partition of the set $\mathbb{P}$ of all primes, that is, $\mathbb{P} = \cup_{i\in I}\sigma_i$ and $\sigma_i \cap \sigma_j = \emptyset$ for all $i \neq j$. Let G be a finite group. We say that G is: *σ-primary* if G is a σ_i-group for some $i \in I$; *σ-soluble* if every chief factor of G is σ-primary.

We say that a set $\mathcal{H} = \{H_1, \dots, H_t\}$ of Hall subgroups of G, where H_i is σ-primary ($i = 1, \dots, t$), is a *complete Hall set of type* σ of G if $(|H_i|, |H_j|) = 1$ for all $i \neq j$ and $\pi(G) = \pi(H_1) \cup \dots \cup \pi(H_t)$. We say that a subgroup A of G is: *σ-subnormal* in G if there is a subgroup chain $A = A_0 \leq A_1 \leq \dots \leq A_n = G$ such that either A_{i-1} is normal in A_i or $A_i/(A_{i-1})_{A_i}$ is σ-primary for all $i = 1, \dots, t$; *σ-permutable* in G if G has a complete Hall set $\mathcal{H}$ of type σ such that $AH^x = H^xA$ for all $x \in G$ and all $H \in \mathcal{H}$.
In our report we discuss some properties of σ-subnormal and σ-permutable subgroups of finite groups.

Sequences of commutative monomial algebras and the Hilbert series of noncommutative algebras

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In this talk we present a generalization of the notion of Hilbert function or growth function of a commutative or noncommutative finitely generated associative algebra. In particular, we study its generating series providing conditions for having the sum of such series as a rational function. We develop also an effective method for computing explicitly this rational function. Our approach is based on commutative structures and algebraic constructions as exact sequences and ideal operations. This implies a new viewpoint in the theory and computation of noncommutative Hilbert series.

Gelfand-Kirillov dimension of relatively free graded algebras

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Let us consider $A := M_n(k)$, the algebra of $n \times n$ matrizes with entries in a field k of characteristic 0. Let G be an abelian group and let us suppose A be G-graded by an elementary G-grading. Then we compute the Gelfand-Kirillov (GK) dimension of its relatively free G-graded algebra in m variables. As a consequence we compute the GK dimension of any verbally prime algebra endowed with its "canonical" $G \times \mathbb{Z}_2$-grading.

On non-perfect minimal non-M_rC-groups

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A group G is said to be a minimax group if it has a finite series whose factors satisfy either the minimal or maximal condition. Let $D(G)$ denotes the subgroup of G generated by all Chermikov divisible normal subgroups of G is a soluble-by-finite minimax group and $D(G) = 1$, then G is said to be a *reduced minimax* group. Also G is said to be an M_rC-group (respectively, $(PF)C$-group), if $G/C_G(x^G)$ is reduced minimax (respectively, polycyclic-by-finite) group for all $x \in G$. These are generalizations of the familiar property of being an FC-group. Finally, if X is a class of groups, then G is said to ve a minimal non-X-groups if it is not an X-group but all of whose proper subgroups are X-groups. Many results have been obtained on minimal non-X-groups, for various choice of X. In particular, in [1] and [2], Belyaev and Sesekin characterized minimal non-FC-groups when they have non-trivial finite or abelian factor group. In [3], we proved that for groups having proper subgroups of finite index, the property of being a minimal non-M_rC-group, a minimal non-$(PF)C$-group and a minimal non FC-group are equivalent. Here we present a similar result for non-perfect groups. Our result is a following

Theorem 4. *Let G be a non perfect group. Then the following conditions are equivalent.*

(i) G is a minimal non-M_rC-group,

(ii) G is a minimal non-$(PF)C$-group,

(iii) G is a minimal non-FC-group.

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The congruence η^* on semigroups

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In this work we define a congruence η^* on semigroups. For the finite semigroups S, η^* is the smallest congruence relation such that S/η^* is a nilpotent semigroup (in the sense of Malcev). In order to study the congruence relation η^* on finite semigroups, we define a **CS**-diagonal finite regular Rees matrix semigroup. We prove that, if S is a **CS**-diagonal finite regular Rees matrix semigroup then S/η^* is inverse. Also, if S is a completely regular finite semigroup, then S/η^* is a Clifford semigroup.
We show that, for every non-null principal factor A/B of S, there is a special principal factor C/D such that every element of $A \setminus B$ is η^*-equivalent with some element of $C \setminus D$. We call the principal factor C/D, the η^*-root of A/B. All η^*-roots are **CS**-diagonal. If certain elements of S act in the special way on the **R**-classes of a **CS**-diagonal principal factor then it is not an η^*-root. Some of these results are also expressed in terms of pseudovarieties of semigroups.

Generalized Commutativity, Hultman numbers and isoclinism of groups

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Let G be a finite group and π be a permutation from S_n. We investigate the distribution of the probabilities of the equality $a_1a_2\cdots a_{n-1}a_n = a_{\pi_1}a_{\pi_2}\cdots a_{\pi_{n-1}}a_{\pi_n}$, when π varies over all the permutations in S_n. The probability $Pr_\pi(G) = Pr(a_1a_2\cdots a_{n-1}a_n = a_{\pi_1}a_{\pi_2}\cdots a_{\pi_{n-1}}a_{\pi_n})$ is identical to $Pr_1^\omega(G)$, with $\omega = a_1a_2...a_{n-1}a_na_{\pi_1}^{-1}a_{\pi_2}^{-1}\cdots a_{\pi_{n-1}}^{-1}a_{\pi_n}^{-1}$, as it is defined by Das and Nath in 2012. The probability of the equality $a_1a_2 = a_2a_1$, for which $n = 2$ and $\pi = \langle 2\ \ 1\rangle$, was computed by Gustafson in 1973. It turns out that these probabilities, for a permutation π, depend only on the number $c(G(\pi))$ of the cycles in the cycle graph $G(\pi)$ of π. The cycle graph of a permutation was introduced by Bafna and Pevzner in 1998. We describe the spectrum of the probabilities of permutation equalities in a finite group as π varies over all the elements of S_n. This spectrum is closely related

to the partition of $n!$ into a sum of the corresponding Hultman numbers. Das and Nath proved in 2012, that $Pr^{\omega_1}(G_1) = Pr^{\omega_1}(G_2)$ for every permutation ω, in case G_1 and G_2 are isoclinic. We show, that the opposite is not necessarily true, which means that there exist G_1 and G_2 of order 64 both, which are not isoclinic, while $Pr^{\omega_1}(G_1) = Pr^{\omega_1}(G_2)$ for every permutation $\omega \in S_n$. We also show a case when G_1 and G_2 are weakly isoclinic, but even the commuting probability ($Pr(ab = ba)$) of G_1 and G_2 have different values. This is a joint work with Y. Cherniavsky, A. Goldstein and V. E. Levit.

Commutative Nilpotent Algebras and Weil Representations

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Let $\mathbb{F}$ be the field $GF(q^2)$ of q^2 elements and let V be an $\mathbb{F}$-vector space endowed with a nonsingular Hermitian form φ. In this joint work with H. N. Ward, we investigate whether the restrictions of the Weil representation of the unitary group $\mathrm{U}(\varphi)$ to certain subgroups are multiplicity-free. These subgroups consist of the members of $\mathrm{U}(\varphi)$ in subalgebras of the form $\mathbb{F}I + N$, where N is a commutative nilpotent subalgebra of $\mathrm{End}_{\mathbb{F}}V$ with the further property that N contains its annihilator.

Non-cancellation group computation

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Consider the semidirect product $G_i = \mathbb{Z}_{n_i} \rtimes_{\omega_i} \mathbb{Z}$. Methods for computation of the non-cancellation groups $\chi(G_1 \times G_2)$ and $\chi(G_i^k)$, $k \in \mathbb{N}$ were developed [3] and in [1] respectively. In this talk, we are going to develop a general method of computing $\chi(G_1 \times G_2, h)$, where $h : F \hookrightarrow G_1 \subseteq G_1 \times G_2$ and F a finite group.

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