Abstracts of short communications

Advances in Group Theory and Applications 2013

June, 10th - 14th, 2013

Hotel Lo Scoglio, Porto Cesareo (Lecce, ITALY)

Cocharacters of bilinear mappings and graded matrices

Stefania Aquè

Dipartimento di Matematica ed Informatica, Università di Palermo, Via Archirafi 34, 90123 Palermo, Italy
E-mail: aque@math.unipa.it

Let $M_k(F)$ be the algebra of $k \times k$ matrices over a field F of characteristic 0. If G is any group, we endow $M_k(F)$ with the elementary grading induced by the k-tuple $(1, \ldots, 1, g)$ where $g \in G$, $g^2 \neq 1$. Then the graded identities of $M_k(F)$ depending only on variables of homogeneous degree g and g^{-1} are obtained by a natural translation of the identities of bilinear mappings. We study such identities by means of the representation theory of the symmetric group. We act with two copies of the symmetric group on a space of multilinear graded polynomials of homogeneous degree g and g^{-1} and we find an explicit decomposition of the corresponding graded cocharacter into irreducibles.

Automorphisms of extremal self-dual codes

Martino Borello

I will present some techniques developed to study the automorphism group of certain binary linear codes, namely the extremal self-dual ones. These techniques involve some modular representation theory and actions of permutation groups on finite combinatorial structures. The usual problem is to find a relatively small set of representatives for the action of a group on a set of codes and then do an exhaustive test on such set with Magma checking properties as the minimum distance and the self-duality. I will present some applications of these method to the study of the automorphism group of the putative self-dual extremal code of length 72, which is a long-standing open problem of classical coding theory.

[1] M. Borello, The automorphism group of a self-dual [72; 36; 16] binary code does not contain elements of order 6, 6, IEEE Transactions on Information Theory 58(12): 7240-7245 (2012) (DOI:10.1109/TIT.2012.2211095);

[2] M. Borello and W. Willems, Automorphism of order 2p in binary self-dual extremal codes of length a multiple of 24, (to appear in IEEE Transactions on Information Theory) (DOI:10.1109/TIT.2013.2243802);

[3] M. Borello, F. Dalla Volta and G. Nebe, The automorphism group of a self-dual [72; 36; 16] code does not contain $\mathcal{S}_3$, $\mathcal{A}_4$ or D_8, arXiv:1303.4899, (submitted)

Symmetric Units and Group Identities in Group Algebras

Victor Bovdi

Let $U(A)$ be the group of units of an algebra A (over a field F) with involution $*$. Let $S_*(A) = \{u \in U(A) \mid u = u^*\}$ be the set of symmetric units of A.

Algebras with involution have been actively investigated. In these algebras there are many symmetric elements, for example: $x + x^*$ and xx^* for an arbitrary element $x \in A$. This raises natural questions about properties of the symmetric elements and symmetric units. One such question is whether the symmetric units satisfy a group identity in the group algebra. A structure theorem of algebras with involution whose symmetric elements satisfy a polynomial identity was obtained earlier in [1].

In [3] we classified the cases when the symmetric units commute (i.e. $S_*(KG)$ satisfies the group identity $f(x, y) = x^{-1}y^{-1}xy$) in the modular group algebra KG of a locally finite p-group G. The solution of this question for integral group rings and for some modular group rings of arbitrary groups over rings was obtained in [4, 5].

A. Giambruno, S.K. Sehgal and A. Valenti in [6] classified the group algebras of torsion groups over infinite fields of odd characteristic, whose symmetric units satisfy a group identity. An extension of this classification was given for the group algebra KG of an arbitrary group G over a field K of odd characteristic in [2]. We continue this investigation for fields of even characteristic.

[1] S. A. Amitsur, Rings with involution, Israel J. Math., 6:99–106, 1968;

[2] V. Bovdi, Symmetric units and group identities in group algebras. I., Acta Math. Acad. Paedagog. Nyhzi. (N.S.) 22(2):149159, 2006;

[3] V. Bovdi, L. G. Kovács, and S. K. Sehgal, Symmetric units in modular group algebras, Comm. Algebra, 24(3):803–808, 1996;

[4] V. Bovdi and M. M. Parmenter, Symmetric units in integral group rings, Publ. Math. Debrecen, 50(3-4):369–372, 1997;

[5] V. Bovdi, On symmetric units in group algebras, Comm. Algebra, 29(12):5411–5422, 2001;

[6] A. Giambruno, S. K. Sehgal, and A. Valenti, Symmetric units and group identities, Manuscripta Math., 96(4):443–461, 1998.

Groups with every subgroup ascendant by finite

Sergio Camp-Mora

We will call H an ascendant-by-finite subgroup of a group G if there exists a subgroup K of H which is ascendant in G and such that the index $\mid H : K \mid$ is finite.

It is proved that a locally finite group with every subgroup ascendant-by-finite is necessarily a finite extension of a locally nilpotent group. As a consequence, it is shown that those groups are a finite extension of a Gruenberg group.

Previously, it had been proved that the locally finite groups with every subgroup normal-by-finite are abelian-by-finite. Moreover locally finite groups with every subgroup permutable-by-finite are modular-by-finite.

On groups with given absolute central factor group

M. Chaboksavar* [1]
Department of Mathematics, Mashhad Branch, Islamic Azad University, Mashhad, Iran

M. Farrokhi D. G.
Department of Pure Mathematics, Ferdowsi University of Mashhad, Mashhad, Iran

Abstract

In this paper we classify the finite groups G whose absolute central factors are cyclic, or isomorphic to $Z_p \times Z_p$ and Q_8.

1. Introduction

The study of groups whose central factor groups are given is an important problem in theory of groups. Baer [1] studied the structure of groups whose central factor groups are abelian. In 1964, Hall and Senior [4] introduced the notion of capable groups. A group G is called *capable* if there exists a group E such that $G \cong E/Z(E)$.

In 1993, Hegarty [6] defined the autocommutator of groups. Let G be a group and $A = Aut(G)$ be the group of automorphisms of G. Then for each $x \in G$ and $\alpha \in A$, $[x, \alpha] = x^{-1}x^{\alpha}$ is the *autocommutator* of x and α. Indeed, the autocommutators are

[1] Young Researchers Club and elites, Mashhad Branch, Islamic Azad University, Mashhad, Iran

a generalization of commutators. We know that for every two elements x and y of G, $[x,y] = x^{-1}y^{-1}xy = xx^{\varphi_y}$, where $x^{\varphi_y} = y^{-1}xy$. So, autocommutator is just a commutator in which inner automorphisms is replaced by an automorphisms. This leads us to generalize the center of groups by means of autocommutators. The set

$$L(G) = \{g \in G : [g,\alpha] = 1, \alpha \in Aut(G)\}$$

is a characteristic central subgroup of G and called the *absolute center* of G.

Recently studying of autocommutatrs and absolute center of groups are interested. In this talk, we classify some finite groups whose absolute central factors have a given structure.

The above remark leads one to suspect that when the structure of absolute central factor $G/L(G)$ is very simple (and by very simple we mean that $G/L(G) \cong < 1 >$ or $G/L(G)$ is cyclic), the structure of G is as follows:

Theorem 1. Assume that G is a finite group such that $G/L(G) \cong < 1 >$. Then it is clear that G is the trivial group or $G \cong Z_2$.

A well-know result in theory of groups states that a group has never a non-trivial cyclic central factor. In more details, there is no nontrivial cyclic capable group. However, the following theorem shows that this is not the case for absolute central factor groups.

Theorem 2. A finite group G is cyclic if and only if $G/L(G)$ is cyclic.

It is clear that in the concept of capable groups, there is no solution for the equation $X/Z(X) \cong G$. For example we can not find a group X such that $X/Z(X) \cong Q_8$. We will show that there is no group G such that $G/L(G) \cong Q_8$.

In the next section, we shall discuss on the groups G such that $G/L(G)$ is isomorphic to $Z_p \times Z_p$ for some prime p and Q_8

2. Main results

Clearly, there are infinitely many groups G such that $G/Z(G) \cong Z_p \times Z_p$ for a given prime p and there is no group such that $G/Z(G) cong Q_8$. Indeed all groups $A \times H$, where A is an abelian group and H is a group such that $H/Z(H) \cong Z_p \times Z_p$ satisfy the mentioned property. The surprising news is that for absolute centers the situation is completely different from that of centers as the following result shows.

Theorem 3. Let G be a finite group such that $G/L(G) \cong Z_p \times Z_p$. Then

$$G \cong \begin{array}{l} Z_p \times Z_p, \\ Z_p \times Z_p \times Z_2, \quad p \text{ is odd}, \\ Z_4 \times Z_2, \\ D_8, \\ Q_8. \end{array}$$

Theorem 4. There is no group G such that $G/L(G) \cong Q_8$.

References

[1] R. Baer, Groups with Abelian central quotient group, *Trans. Amer. Math. Soc.* **44**(3) (1938), 357–386.;

[2] J. N. S. Bidwell, M. J. Curran and D. J. McCaughan, Automorphisms of direct products of finite groups, *Arch. Math.* **86** (2006), 481–489;

[3] M. Farrokhi D. G. and M. R. R. Moghaddam, On the center of automorphism group of a group, *submitted.*;

[4] M. Hall and J. K. Senior, *The groups of order* $2n$ *(*$n \leq 6$*)*, MacMillan, New York, 1964;

[5] P. Hall, The classification of prime-power groups, *J. Reine Angew. Math.* **182** (1940), 130–141;

[6] P. V. Hegarty, Autocommutator subgroups of finite groups, *J. Algebra* **190** (1997), 556–562.

Incidence Matrices, Invariant Factors and Orbits of a permutation group

Francesca Dalla Volta

Let $0 \leq t, k \leq n$ be integers with $t + k \leq n$ and let Ω be a set of size n. We denote by $M^n_{t,k}$ the Incidence Matrix of t- versus k-subsets of Ω; $M^n_{t,k}$ is the matrix with rows indexed by t-subsets x, columns indexed by k-subsets y and (x, y)-entry equal to 1 if $x \subseteq y$ or $x \supseteq y$, and equal to 0 otherwise. In this talk I will be interested in some results obtained together with Johannes Siemons about invariant factors of this matrix (that is the Smith form of it) and some applications to Designs and to Orbits of a Permutation Group on Ω.

On the ring of inertial endomorphisms of an abelian group

Ulderico Dardano and Silvana Rinauro

In the study of soluble groups with many inert subgroups (see [2], [4]) it seems relevant the consideration of inertial endomorphisms (see [1], [3]) of an abelian group $(A, +)$, that is endomorphisms φ with the property:

$$\forall X \leq A \quad |\varphi(X) + X/X| < \infty.$$

Clearly all "multiplications" and finitary endomorphisms have this property. A less trivial example of inertial endomorphism is $0 \oplus \frac{1}{2}$ on $A = \mathbb{Z}(2)^{\omega} \oplus \mathbb{Q}_2$. More complicated examples will be exhibited.

In this talk we describe the ring of inertial endomorphisms and consider the group of invertible ones.

[1] U. Dardano, S. Rinauro, *Inertial automorphisms of an abelian group*, Rend. Sem. Mat. Univ. Padova 127 (2012) 213-233;

[2] M. De Falco, F. de Giovanni, C. Musella, N. Trabelsi, *Strongly inertial groups*, to appear on Comm. Algebra;

[3] D. Dikranjan, A. Giordano Bruno, L. Salce, S. Virili, *Fully inert subgroups of divisible Abelian groups*, to appear on Journal Group Theory;

[4] D.J.S. Robinson, *On inert subgroups of a group*, Rend. Sem. Mat. Univ. Padova 115 (2006), 137-159.

Groups with finiteness conditions on subgroups of inifnte rank

Maria De Falco

A group G is said to have*finite Prüfer rank r* if every finitely generated subgroup of G can be generated by at most r elements, and r is the least positive integer with such property. We discuss here the imposition of certain embedding properties to the subgroups of infinite rank of a soluble group.

Maximal subgroups of finite groups and Sylow permutability [2]

Ramon Esteban-Romero [3]

In this talk we will consider only finite groups. We say that a subgroup H of a group G permutes with a subgroup K of G if HK is a subgroup of G. The subgroup H is said to be permutable (respectively, S-permutable) if it permutes with all subgroups (respectively, all Sylow subgroups) of G. Finite groups in which permutability (respectively, S-permutability) is a transitive relation are called PT-groups (respectively, PST-groups). The classes of PT-groups and PST-groups, together with the class of T-groups or groups in which normality is transitive, have been object of an extensive study, with many characterisations available. Kaplan [1] presented some new characterisations of soluble T-groups in terms of maximal subgroups. The aim of this talk is to present PT- and PST-versions of Kaplan's results in terms of the containment of non-permutable (respectively, non-S-permutable) subgroups in non-normal maximal subgroups. This enables a better understanding of the relations between these classes.

The results of this talk appear in [2].

Mathematics Subject Classification (2010): 20D05, 20D10, 20E15, 20E28, 20F16
Keywords: finite groups, permutability, Sylow-permutability, maximal subgroups, supersolubility.

[1] A. Ballester-Bolinches, J. C. Beidleman, R. Esteban-Romero, and V. Pérez-Calabuig, Maximal subgroups and PST-groups, Cent. Eur. J. Math., 11(6):1078–1082, 2013.;

[2] G. Kaplan, On *T*-groups, supersolvable groups, and maximal subgroups, Arch. Math. (Basel), 96(1):19–25, 2011.

Computing generators of the unit group of an integral abelian group ring

Paolo Faccin

We describe an algorithm for obtaining generator of the unit group of the integral group ring $\mathbb{Z}G$ of a finite abelian group G. We used our implementation in MAGMA

[2]This work has been supported by the research grant MTM2010-19983-C03-01 from MICINN, Spain.

[3]Institut Universitari de Matemàtica Pura i Aplicada, Universitat Politècnica de València, Camí de Vera, s/n, 46022 València, Spain, email: `resteban@mat.upv.es`; current address: Departament d'Àlgebra, Universitat de València, Dr. Moliner, 50, 46100 Burjassot, València, Spain, email: `Ramon.Esteban@uv.es`

of this algorithm to compute the unit groups of $\mathbb{Z}G$ for G of order up to 110. In particular for those cases we obtained the index of the group of Hoechsmann units in the full unit group. At the end of the paper we describe an algorithm for more general problem of finding generators of an arithmetic group corresponding to a diagonalisable algebraic group.

(Joint work with Willem De Graaf and Wilhelm Plesken, appeared in Journal of Algebra 373, 441-373, 2013)

Group rings of finite strongly monomial groups: central units and primitive idempotents

Inneke Van Gelder

Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussels, Belgium

(joint work with Eric Jespers, Gabriela Olteanu and Ángel del Río)

For a finite group G we denote by $\mathcal{U}(\mathbb{Z}G)$ the unit group of the integral group ring $\mathbb{Z}G$. Its group of central units is denoted by $\mathcal{Z}(\mathcal{U}(\mathbb{Z}G))$.

In [1] we have proved that the group generated by the so-called generalized Bass units contains a subgroup of finite index in $\mathcal{Z}(\mathcal{U}(\mathbb{Z}G))$ for any arbitrary finite strongly monomial group G. No multiplicatively independent set of generators for such a subgroup was obtained. We call such a set a virtual basis. However, we obtained an explicit description of a virtual basis of $\mathcal{Z}(\mathcal{U}(\mathbb{Z}G))$ when G is a finite abelian-by-supersolvable group (and thus a strongly monomial group) such that every cyclic subgroup of order not a divisor of 4 or 6 is subnormal in G. We present an extension of these results on the construction of a virtual basis of $\mathcal{Z}(\mathcal{U}(\mathbb{Z}G))$ to a class of finite strongly monomial groups containing the metacyclic groups $G = C_{q^m} \rtimes C_{p^n}$ with p and q different primes and C_{p^n} acting faithfully on C_{q^m}. Our proof makes use of strong Shoda pairs and the description of the Wedderburn decomposition of $\mathbb{Q}G$ obtained by Olivieri, del Río and Simón in [2].

In [3] a complete set of matrix units (and in particular, of orthogonal primitive idempotents) of each simple component in the rational group algebra $\mathbb{Q}G$ is described for finite nilpotent groups G. As an application one obtains a factorization of a subgroup of finite index of $\mathcal{U}(\mathbb{Z}G)$ into a product of three nilpotent groups, and one explicitly constructs finitely many generators for each of these factors. We present a description of a complete set of matrix units for a class of finite strongly monomial groups containing the finite metacyclic groups $C_{q^m} \rtimes C_{p^n}$ with C_{p^n} acting faithfully on C_{q^m}. For the latter groups we obtain as an application of these results (and the earlier results on central units) again an explicit construction of finitely many generators of three nilpotent subgroups that together generate a subgroup of finite index in $\mathcal{U}(\mathbb{Z}G)$.

[1] E. Jespers; Á del Río; G. Olteanu; I. Van Gelder, *Central units of integral group rings*, Proc. Amer. Math. Soc. (in press);

[2] A. Olivieri; Á. del Río; J. J. Simón, *On monomial characters and central idempotents of rational group algebras*, Communications in Algebra 32 (2004), no. 4, 1531 - 1550;

[3] HE. Jespers; G. Olteanu; Á. del Río, *Rational group algebras of finite groups: from idempotents to units of integral group rings*, Algebr. Represent. Theory 15 (2012), no. 2, 359 - 377.

Some results on tensor nilpotent groups

Elaheh Mohammadzadeh

Department Of Mathematics, Faculty Of Science, Payame Noor University, 19395-3697, Tehran, Iran

Hanieh Golmakani

Department Of Mathematics, Faculty Of Science, Mashhad Branch, Islamic Azad University, Mashhad, 91735-413, Iran

Tensor analogues of right 2-Engel elements in groups were introdused by Biddle and Kappe. P.Moravec presented some results on tensor analogues of 2-Engel groups. In this paper first we introduce the concept of tensor nilpotent groups, then we see in $3_{\otimes}$-Engle groups, $< x, x^y >$ is tensor nilpotent of class at most 2, for all $x, y \in G$. Also with additional conditions we prove that if $y^x = a$ then $\langle a, a^y \rangle$ is tensor nilpotent of class 2.

Keyword and phrases: nilpotent, Engel group, non-abelian tensor product.

[1] D.P. Biddle, L.C. Kappe R.F. Morse, On subgroups related to the tensor center. Glasg. Math. J. 42 (2003). no 2, 323-332;

[2] R. Brown, J.-L. Loday, Van Kampen theorems for diagrams of spaces. Topology 26 (1987), no. 3, 311-335;

[3] G. Ellis , Tensor products and q-crossed modouls, J. London Math. Soc. (2) 51 (1995), 243-258;

[4] A. McDermott, The non abelian tensor product of group: computations and structural results, Department of Mathematics facuality of arts national university of ireland galway. 1998;

[4] Shaban Sedghi, avalable at http://www.math.kth.se/4ecm/abstracts/3.6.pdf;

[5] A. Abdollahi, Left 3-Engel elements in groups. Journal of Pure and Applied Algebra. (2004), 1-6.

On Lewin's problem

Jairo Zacarias Goncalves

University of Sao Paulo, Brazil

We address the following question, attributed to J. Lewin: Let kG be the group algebra of the nonabelian torsion free nilpotent group G, and let $D = Q(kG)$ be its field of fractions. If x and y are two noncommuting elements of G, then $1 + x$ and $1 + y$ generate a free noncyclic subgroup in D.

We start recalling some background, and a former solution obtained by Goncalves, Mandel and Shirvani in J. Algebra, 1999. We finish giving a recent extension, due to Goncalves and Passman.

Weyl groups of fine gradings on $\mathfrak{e}_6$

Valerio Guido

Given the Cartan decomposition of a semisimple complex Lie algebra, the automorphism group of its root system is the so-called extended Weyl group that generalizes the well-known Weyl group generated by reflections through the hyperplanes orthogonal to the roots. Because of the usefulness of such a decomposition in studying the properties of the algebraic structure, some other partitions of Lie algebras have been investigated together with their automorphisms, but it was in 1989 that a systematic study of Lie gradings started in an article by J. Patera and H. Zassenhaus [1].

Let L be a Lie algebra over an algebraically closed field of characteristic zero and let G be an abelian group. A G-grading on L is a decomposition into subspaces $\Gamma : L = \oplus_{g \in G} L_g$ such that $L_g L_h \subset L_{g+h}$ for all $g, h \in G$. The Weyl group of Γ is the quotient of two subgroups of $Aut(L)$: the subgroup generated by the automorphisms that permute the components L_g and the subgroup of the automorphisms that stabilize them. The Cartan decomposition is a very special fine grading (i.e., it cannot be refined) whose Weyl group coincides with the classical extended Weyl group of the roots. The aim of this talk is to describe the Weyl groups of fine gradings of the exceptional Lie algebra $\mathfrak{e}_6$. These gradings have been recently classified by C. Draper and A. Viruel in [2].
This is a work in preparation jointly with D. Aranda and C. Draper.

[1] J. Patera and H. Zassenhaus, *On Lie gradings. I*, Linear Algebra and its Applications **112** (1989), 87 – 159;

[2] C. Draper and A. Viruel, *Fine gradings on* $\mathfrak{e}_6$ preprint arXiv: 1207.6690v1 .

Discontinuous actions on $\mathbb{H}^2 \times \mathbb{H}^2$

Ann Kiefer

The main goal is the investigation on the unit group of an order $\mathcal{O}$ in a rational group ring $\mathbb{Q}G$ of a finite group G. In particular we are interested in the unit group of $\mathbb{Z}G$. For many finite groups G a specific finite set B of generators of a subgroup of finite index in $\mathcal{U}(\mathbb{Z}G)$ has been given. The only groups G excluded in this result are those for which the Wedderburn decomposition of the rational group algebra $\mathbb{Q}G$ has a simple component that is either a non-commutative division algebra different from a totally definite quaternion algebra or a 2×2 matrix ring $M_2(D)$, where D is either $\mathbb{Q}$, a quadratic imaginary extension of $\mathbb{Q}$ or a totally definite rational division algebra $\mathcal{H}(a; b; \mathbb{Q})$.

In some of these cases, up to commensurability, the unit group acts discontinuously on a direct poduct of hyperbolic 2- or 3-spaces. The aim is to generalize the theorem of Poincaré on fundamental domains and group presentations to these cases. For the moment we have done this for the Hilbert Modular Group, which acts on $\mathbb{H}^2 \times \mathbb{H}^2$, in joint work with Á. del Río, E. Jespers.

The Automorphism Group of Some Matrix Rings

Feride Kuzucuoğlu

Department of Mathematics, University of Hacettepe , 06532 Beytepe,Ankara, Turkey
E-mail: feridek@hacettepe.edu.tr

Let K be an associative ring with identity and let $NT(n,K)$ be the ring of all lower nil triangular $n \times n$ matrices over K. A ring R is called a radical (Jacobson) ring if $(R, \circ)$ is a group (adjoint group) with respect to $a \circ b = a + b + ab$. Let $AutR$ be the automorphism group of any radical ring R. It coincides with the intersection of the automorphism group of the adjoint group $G(R)$ and the automorphism group of the associated Lie ring $\Lambda(R)$ of R.

For arbitrary associative ring K with identity automorphism groups of $NT_n(K)$, $G(NT_n(K))$ and $\Lambda(NT_n(K))$ were described by Levchuk. Let $M_n(J)$ be the ring of all $n \times n$ matrices over an ideal J of K and

$$R_n(K,J) := NT_n(K) + M_n(J)$$

Our aim is to describe the automorphism group $AutR_n(K,J)$ for arbitrary K and quasi-regular ideal J and derivations of $R_n(K,J)$.

Varieties of algebras of polynomial growth

Daniela La Mattina

Let A be an associative algebra over a field F of characteristic zero and let $c_n(A)$, $n = 1, 2, \ldots$, be its sequence of codimensions. It is well known that the sequence of codimensions of a PI-algebra either grows exponentially or is polynomially bounded.

In this note we are interested in the case of polynomial growth. For this case a celebrated theorem of Kemer characterizes the algebras whose sequence of codimensions is polynomially bounded as follows. Let G be the infinite dimensional Grassmann algebra over F and let UT_2 be the algebra of 2×2 upper triangular matrices. Then $c_n(A)$, $n = 1, 2, \ldots$, is polynomially bounded if and only if G, $UT_2 \notin \mathrm{var}(A)$, where $var(A)$ denotes the variety of algebras generated by A.

In the setting of G-graded algebras, where G is a finite group, the sequence of graded codimensions is polynomially bounded if and only if $\mathrm{var}^{gr}(A)$ does not contain a finite list of G-graded algebras. The list consists of group algebras of groups of order a prime number, the infinite dimensional Grassmann algebra and the algebra of 2×2 upper triangular matrices with suitable gradings. Such algebras generate the only varieties of G-graded algebras of almost polynomial growth, i.e., varieties of exponential growth such that any proper subvariety grows polynomially.

In this note, we completely classify all subvarieties of the G-graded varieties of almost polynomial growth by giving a complete list of finite dimensional G-graded algebras generating them.

Letterplace approach to (group) algebras

Roberto La Scala

Let $A = \langle x_1, \ldots, x_n | \; r_1 = 0, \ldots, r_m = 0 \rangle$ be a finitely presented (noncommutative) algebra, say the group algebra KG of a finitely presented group. If one wants to know if A is finite dimensional (G is finite) or to compute its growth, GKdim, etc, one approach consists in computing a Gröbner basis of the two-sided ideal $I = \langle r_1, \ldots, r_m \rangle$ of the free associative algebra $K \langle x_1, \ldots, x_n \rangle$ and then to count in the set of normal words. A next step may consist in applying the "letterplace correspondence", introduced in [1, 2] for the graded case, that transforms I (and related computations) into a N-invariant ideal J of the free commutative $\mathbb{N}$-algebra $K \{x_1, \ldots, x_n\}$, that is a free object in the category of finitely generated commutative algebras endowed with the action (by endomorphisms) of the monoid $(\mathbb{N}, +)$ of natural numbers. In other words, we can modelize a finitely presented (non-commutative) algebra $A = K \langle x_1, \ldots, x_n \rangle / I$ by means of a finitely presented commutative $\mathbb{N}$-algebra $K \{x_1, \ldots, x_n\} / J$. In this talk we expose these ideas with special emphasis to the generalization obtained in [3] of the letterplace correspondence for non-graded ideals that especially is the case of group algebras.

[1] La Scala, R.; Levandovskyy, V., Letterplace ideals and non-commutative Gröbner bases. J. Symbolic Comput., 44 (2009), 1374-1393.

[2] La Scala, R.; Levandovskyy, V., Skew polynomial rings, Gröbner bases and the letterplace embedding of the free associative algebra, J. Symbolic Comput., 48 (2013), 110-131.

[3] La Scala, R., Extended letterplace correspondence for nongraded noncommutative ideals and related algorithms, preprint (2012), 1-20. arXiv:1206.6027.

A restriction on centralizers in finite groups

Gustavo A. Fernández-Alcober, Leire Legarreta, Antonio Tortora, and Maria Tota

For a given $m \geq 1$, we consider the finite non-abelian groups G for which $|C_G(g) : \langle g \rangle| \leq m$ for every $g \in G \setminus Z(G)$. We show that the order of G can be bounded in terms of m and the largest prime divisor of the order of G. Our approach relies on dealing first with the case where G is a non-abelian finite p-group. In that situation, if we take $m = p^k$ to be a power of p, we show that $|G| \leq p^{2k+2}$ with the only exception of Q_8. This bound is best possible, and shows that the order of G can be bounded by a function of m alone in the case of nilpotent groups.

Matematika Saila, Euskal Herriko Unibertsitatea UPV/EHU, Bilbao, Spain
E-mail address: gustavo.fernandez@ehu.es

Matematika Saila, Euskal Herriko Unibertsitatea UPV/EHU, Bilbao, Spain
E-mail address: leire.legarreta@ehu.es

Dipartimento di Matematica, Università di Salerno, Salerno, Italy
E-mail address: mtota@unisa.it

Dipartimento di Matematica, Università di Salerno, Salerno, Italy
E-mail address: antortora@unisa.it

Ekedahl invariants for finite groups

Ivan Martino

In 2009 T. Ekedahl introduced some cohomological invariants for affine finite groups of finite type over the algebraically closed field k of characteristic zero.
These relate, naturally, to invariant theory for groups and also to the Noether Problem (one wonders about the rationality of the extension $F(x_g : g \in G)^G$ over F, for a field F and a finite group G).
In this talk, we introduce these invariants, we state the literature results and we show that these invariants are trivial for every finite group in $Gl_3(\mathbb{C})$ and for the fifth discrete Heisenberg group H_5.

On certain weak Engel-type conditions in groups

Maurizio Meriano

Let $w(x, y)$ be a word in two variables and $\mathcal{W}$ the variety of groups determined by w. In this talk, based on joint work with C. Nicotera [1], we discuss the following problem: if for every pair of elements a, b in a group G there exists $g \in G$ such that $w(a^g, b) = 1$, under what conditions does the group G belong to $\mathcal{W}$? In particular, we focus on the n-Engel word $w(x, y) = [x,_n y]$.

[1] M. Meriano, C. Nicotera, *On certain weak Engel-type conditions in groups*, to appear.

Cleanness of group algebras

Paula Murgel Veloso

In collaboration with Prof. Álvaro Raposo, Universidad Politécnica de Madrid.

An element of a ring is clean if it is the sum of a unit and an idempotent. A ring is clean if every element in it is clean. If the representation of an element in this way is unique, the element is said to be uniquely clean, and the ring is uniquely clean if all its elements are so.
The property of cleanness was formulated by Nicholson [1] in the course of his study of exchange rings, for both are closely related: clean rings are always exchange rings and the converse is true when idempotents are central in the ring. Uniquely clean rings were studied later [2] showing that they are a sort of generalization of Boolean rings.
In the realm of group rings these properties have been studied with the aim to characterize the rings R and groups G such the group ring RG is clean or uniquely clean.
At first the focus was set on uniquely clean group rings [3], which is a quite restrictive property and leaves no much room for groups and rings. For instance, a necessary condition for RG to be uniquely clean is R to be itself uniquely clean and G to be a 2-group. This condition is also sufficient if the group is taken among locally finite groups or solvable groups.
Recently Wang and You [4] studied the property of cleanness of group rings getting nice results when the ring of coefficients R is commutative and the group G is a p-group. If p is in the Jacobson radical of R, then RG is clean if and only if R is

clean.
In this communication we show results about the cleanness property in group algebras KG, where K is a field. But there is a key general result about clean rings of Camillo and Yu [5] in which they establish that semiperfect rings without an infinite set of orthogonal idempotents are clean rings. Therefore, since every finite dimensional K algebra is in this situation, it is clean, and we must go to infinite dimensional group algebras to find examples which are not clean.

[1] W.K. Nicholson, Lifting idempotents and exchange rings, Trans. Amer. Math. Soc. 229 (1977), 269-278.

[2] W.K. Nicholson; Y. Zhou, Rings in which elements are uniquely the sum of an idempotent and a unit, Glasg. Math. J. 46 (2004), 227-236.

[3] J. Chen; W.K. Nicholson; Y. Zhou, Group rings in which every element is uniquely the sum of a unit and an idempotent, Journal of Algebra 306 (2006), 453-460.

[4] X. Wang; H. You, Cleanness of the group ring of an Abelian p-group over a commutative ring, Alg. Colloq. 19 (2012), 539-544.

[5] V.P. Camillo; H.P. Yu, Exchange rings, units and idempotents, Comm. Alg. 22 (1994), 4737-4749.

Groups with large systems of normal subgroups

Carmen Musella

Groups in which the condition of being normal is imposed to the members of a system of subgroups have been studied by several authors. In particular, the system of all subnormal subgroups and that of all non-abelian subgroups are of relevant interest in the theory of soluble groups. In this context, we discuss the behavior of soluble groups of infinite rank.

On H−kernels of finite semigroups

Vicente Pérez-Calabuig

Given $\mathbf{H}$ a variety of finite groups, that is, a subgroup-closed formation, we can consider the class $\mathbf{S(H)}$ of all $\mathbf{H}-$soluble groups, i.e the groups G in which the trivial subgroup is $\mathbf{H}-$subnormal in G. This class turns out to be important in semigroup theory for a better understanding of the kernel of a finite semigroup relative to $\mathbf{H}$, which plays an important role in the decidability of Mal'cev products. Our aim here is to study the kernel and its corresponding class of soluble semigroups in order to obtain results that clarify its computability.

Jordan Nilpotency in Group Rings

César Polcino Milies

The *Lie bracket* of an associative algebra A is the ring commutator $[x, y] = xy - yx$. Using this bracket to define a new product in A, it becomes a Lie Algebra. This algebra is *nilpotent* of index $n \geq 2$ if this is the smallest positive integer such that $[\dots[[x_1, x_2], x_3], \dots, x_n] = 0$ for all $x_1, x_2, \dots, x_n \in A$.

Let $A = RG$ denote the group ring of a group G over a commutative ring R, with unity. Further, assume that $\alpha \mapsto \alpha^*$ is an involution on RG which is a linear extension of an involution in G. Then, the set $A^- = \{a \in A \mid a^* = -a\}$ of *skew-symmetric* elements of A is a Lie subring.

Similarly, if we define in A a product by $x \circ y = xy + yx$, then it becomes a Jordan ring and the set $A^+ = \{a \in A \mid a^* = a\}$ of *symmetric* elements is a Jordan subring of A. A is called *Jordan nilpotent* of index $n \geq 2$ if n is the smallest positive integer such that $(\dots((x_1 \circ x_2) \circ x_3) \cdots) \circ x_n = 0$ for all $x_1, x_2, \dots, x_n \in A$.

Questions such as when is A Lie nilpotent or when is A^- Lie nilpotent have been discussed in recent literature [1, 2, 5, 6].

We shall discuss similar questions for Jordan nlpotency. This is joint work with E.G. Goodaire.

[1] A. Giambruno and S. K. Sehgal, *Lie nilpotence of group rings*, Comm. Algebra **21** (1993), 4253–4261.

[2] A. Giambruno and S. K. Sehgal, *Group algebras whose Lie algebra of skew-symmetric elements is nilpotent*, Contemp. Math. **420** (2006), 113–120.

[3] Edgar G. Goodaire and César Polcino Milies, *Involutions and anticommutativity in group rings*, *Bull. Canad. Math. Soc.*, **56** (2013), 344-353.

[4] Edgar G. Goodaire and César Polcino Milies, *Jordan Nilpotency in Group Rings*, preprint.

[5] G. T. Lee, *Group rings whose symmetric elements are Lie nilpotent*, Proc. Amer. Math. Soc. **127** (1999), no. 11, 3153–3159.

[6] Gregory T. Lee, S. K. Sehgal, and E. Spinelli, *Lie properties of symmetric elements in group rings II*, J. Pure Appl. Algebra **213** (2009), 1173–1178.

On the unit group of a commutative group ring

Mohamed A. Salim

Let $V(\mathbb{Z}_{p^e}G)$ be the group of normalized units of the group algebra $\mathbb{Z}_{p^e}G$ of a finite abelian p-group G over the ring $\mathbb{Z}_{p^e}$ of residues modulo p^e with $e \geq 1$. The abelian p-group $V(\mathbb{Z}_{p^e}G)$ and the ring $\mathbb{Z}_{p^e}G$ are applicable in coding theory, cryptography and threshold logic (see [1, 4, 5, 7]).
In the case when $e = 1$, the structure of $V(\mathbb{Z}_pG)$ has been studied by several authors (see the survey [2]). The invariants and the basis of $V(\mathbb{Z}_pG)$ has been given by B. Sandling (see [6]). In general, $V(\mathbb{Z}_{p^e}G) = 1 + \omega(\mathbb{Z}_{p^e}G)$, where $\omega(\mathbb{Z}_{p^e}G)$ is the augmentation ideal of $\mathbb{Z}_{p^e}G$. Clearly, if $z \in \omega(\mathbb{Z}_{p^e}G)$ and $c \in G$ is of order p, then $c + p^{e-1}z$ is a nontrivial unit of order p in $\mathbb{Z}_{p^e}G$. We may raise the question whether the converse is true, namely does every $u \in V(\mathbb{Z}_{p^e}G)$ of order p have the form $u = c + p^{e-1}z$, where $z \in \omega(\mathbb{Z}_{p^e}G)$ and $c \in G$ of order p?
We obtained a positive answer to this question and applied it for the description of the group $V(\mathbb{Z}_{p^e}G)$ (see [3]). Our research can be considered as a natural continuation of Sandling's results.

[1] N.N. Aizenberg, A.A. Bovdi, E.I. Gergo, and F.E. Geche. Algebraic aspects of threshold logic. *Cybernetics*, 2:26–30, 1980.

[2] A.A. Bovdi. The group of units of a group algebra of characteristic p. *Publ. Math. Debrecen*, 52(1-2):193–244, 1998.

[3] V.A. Bovdi, M.A. Salim. On the unit group of a commutative group ring. *Submitted for publication*, p.1–10, 2012.

[4] B. Hurley and T. Hurley. Group ring cryptography. *Int. J. Pure Appl. Math.*, 69(1):67–86, 2011.

[5] T. Hurley. Convolutional codes from units in matrix and group rings. *Int. J. Pure Appl. Math.*, 50(3):431–463, 2009.

[6] R. Sandling. Units in the modular group algebra of a finite abelian p-group. *J. Pure Appl. Algebra*, 33(3):337–346, 1984.

[7] W. Willems. A note on self-dual group codes. *IEEE Trans. Inform. Theory*, 48(12):3107–3109, 2002.

Department of Math. Sciences, UAE University - Al-Ain, United Arab Emirates
E-mail address: MSalim@uaeu.ac.ae

On the p-length of a hyperfinite group

Francesca Spagnuolo

Let p be a prime. We say that a periodic p-group P determines the p-length locally if a periodic group G has p-length less or equal then 1 provided that G has a pronormal Sylow subgroup isomorphic to P. The aim of this talk is to analyze hyperfinite groups with Sylow p-subgroups that determine p-length locally in finite groups.

On locally graded groups with a word whose values are Engel

Antonio Tortora

Dipartimento di Matematica, Università di Salerno, Italy
E-mail: antortora@unisa.it

Following Zelmanov's solution of the restricted Burnside problem [3, 4], Wilson showed that any residually finite n-Engel group is locally nilpotent [2]. Recently, in [1], a stronger result was obtained. Namely, it was proved that given positive integers m, n and a multilinear commutator word v, if G is a residually finite group in which all values of the word $w = v^m$ are n-Engel, then the verbal subgroup $w(G)$ corresponding to w is locally nilpotent. In this talk we examine the question whether this is true in the case where G is locally graded rather than residually finite.

[1] R. Bastos, P. Shumyatsky, A. Tortora and M. Tota *On groups admitting a word whose values are Engel*, Internat. J. Algebra Comput. **23** (2013), no. 1, 81–89.

[2] J. S. Wilson, *Two-generator conditions for residually finite groups*, Bull. London Math. Soc. **23** (1991), 239–248.

[3] E. I. Zelmanov, *Solution of the restricted Burnside problem for groups of odd exponent*, Math. USSR-Izv. **36** (1991), 41–60.

[4] E. I. Zelmanov, *Solution of the restricted Burnside problem for* 2*-groups*, Math. USSR-Sb. **72** (1992), 543–565.

On Certain Applications of the Khukhro-Makarenko Theorem

Nadir Trabelsi

Laboratory of Fundamental and Numerical Mathematics
University Setif 1, Setif - Algeria

This is a joint work with Ahmet Arikan and Howard Smith.

A recent result of Khukhro-Makarenko states that if G is a group having a subgroup of finite index which belongs to the variety $\mathfrak{X}_\omega$ defined by an outer commutator word ω, then G has a characteristic subgroup of finite index which belongs to $\mathfrak{X}_\omega$. Here we use this theorem to obtain generalizations of some well-known results. In particular, we prove that if G is a torsion-free locally nilpotent group whose proper subgroups are in $\mathfrak{X}_\omega\mathfrak{T}$, then $G \in \mathfrak{X}_\omega$, where $\mathfrak{T}$ denotes the class of periodic groups; a well known result when $\mathfrak{X}_\omega$ is the variety of nilpotent groups of class at most $c > 0$. We also prove some results on groups in which every proper subgroup is in the class $\mathfrak{X}_\omega\mathfrak{F}$ or $\mathfrak{X}_\omega\mathfrak{C}$, where $\mathfrak{F}$ and $\mathfrak{C}$ denote the classes of finite and Chernikov groups respectively.

Looking at infinite groups through the mirror of their conjugacy classes

Marco Trombetti

The influence of proper subgroups on a group is usually strong. A further evidence of this phenomenon is given here, looking at the behaviour of commutators of proper subgroups of an infinite (generalized) soluble group.