Abstracts

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Trifactorizable groups

B. Amberg

In the investigation of factorized groups one often has to consider groups G of the form $G = AB = AC = BC$ where A, B and C are subgroups of G. It is therefore of some interest to study such trifactorized groups independently and for their own sake. Around 1960 O.H. Kegel considered finite groups $G = AB$, where the subgroups A and B are nilpotent. Among others he proved that G is nilpotent, if the third subgroup C is nilpotent. Later it was shown that G is contained in some saturated formation F containing all nilpotent subgroups, if the subgroup C is an F-group. Here some recent extensions of these results are presented, in particular for soluble groups with minimum condition and more general group classes.

The structure of thin Lie algebras up to the second diamond

M. Avitabile

Thin Lie algebras are graded Lie algebras $L = \bigoplus_{i=1}^{\infty} L_i$ with $\dim L_i \leq 2$ for all i, and satisfying a more stringent but natural narrowness condition modeled on an analogous condition for pro-p groups. The two-dimensional homogeneous components of L, which include L_1, are named *diamonds*. Infinite-dimensional thin Lie algebras with various diamond patterns have been produced, over fields of positive characteristic, as *loop algebras* of suitable finite-dimensional simple Lie algebras, of classical or of Cartan type depending on the location of the second diamond. In this talk we present a description of the initial structure of a thin Lie algebra up to the second diamond, obtained in joint work with G. Jurman and S. Mattarei. Specifically, if L_k is the *second diamond* of L, then the quotient L/L^k is a graded Lie algebras of maximal class. In odd characteristic p, the quotient L/L^k is known to be metabelian, and hence uniquely determined up to isomorphism by its dimension k, which ranges in an explicitly known set of possible values: 3, 5, a power of p, or one less than twice a power of p. However, the quotient L/L^k need not be metabelian in characteristic two. We describe all the possibilities for L/L^k up to isomorphism in the case of even characteristic. In particular, we obtain that $k + 1$ equals a power of two.

Soluble products of finite groups

J. Cossey

Suppose A and B are subgroups of a group G. We say that G is the product of A and B if $G = AB = \{ab : \ a \in A, b \in B\}$. A natural question to ask is whether properties of G can be deduced from properties of A and B. There is an extensive literature on this question. Many properties have been considered- see for example the book of Amberg, Franciosi and de Giovanni- and further restrictions on the products have also been considered. I will concentrate on one particular property. Suppose that G is soluble. Then A and B are certainly soluble but A and B soluble is not enough to ensure that G is soluble and so we may ask the following questions:

What further conditions on A and B will
ensure that $G = AB$ is soluble?
If G is soluble, can we bound the derived length $d(G)$ of G in terms of invariants of A and B?
If $d(G)$ is bounded, can we find the best possible bound?

I will concentrate on the second and third questions. There is quite a lot known about possible bounds but except in some special cases very little is known about best possible bounds. I will survey what is known and discuss some of the problems that we still know little about.

Monotone p-groups

E. Crestani

In the paper *The number of generators of finite p-groups*, A.Mann introduces two classes of p-groups: the class of the monotone p-groups and the class M_s. We recall the definition (here $d(G)$ is the number of generators of G): a p-group G is said to be in M_s, where $1 \leq s \leq p$, if it satisfies the following condition if $H \leq K \leq G$, $KH = p$ and K is not cyclic then $d(H) - 1 < s(d(K) - 1)$. Also, a p-group is said to be monotone if for every H and K with $H \leq K$, we have that $d(H) \leq d(K)$.
We point out that the origin of these classes dates back to 1985, when, during a Saint Andrews Conference, Mann gave a talk on this subject. Mann studies and classifies completely, except for some uncertainty for exponent p^2, the monotone p-groups when $p \neq 2$. During this talk, we present the complete classification of finite monotone 2-groups.

Some results on the probability of mutually commuting n-tuples and n-th nilpotency degree of groups

A. Erfanian

Let G be a finite group, then the probability that a randomly chosen pair of elements of G commutes is defined to be the number of pairs $(x, y) \in G \times G = G^2$ with $xy = yx$ divided to $|G|^2$. Two usual ways to generalize this probability is to consider n-tuples $(x_1, x_2, \ldots, x_n)$ of elements in group G with the property that $x_i x_j = x_j x_i$ for all $1 \leq i, j \leq n$ or $(n+1)$-tuples $(x_1, x_2, \ldots, x_{n+1})$ such that $[x_1, x_2, \ldots, x_{n+1}] = 1$. We denote them by $Pr_n Com(G)$ and $d^{(n)}(G)$, and are called the probability of mutually commuting n-tuples and n-th nilpotency degree of G, respectively. In this talk, we will give some results concerning the above two probabilities and state some relations between these and the notion of the isoclinism. Furthermore, we give one more way of generalization of the probability for infinite groups at the end.

Polynomial Identities and Representations of the Symmetric Group

A. Giambruno

The polynomial identities satisfied by an algebra over a field of characteristic zero are studied through the theory of Young diagrams. One associates to an algebra A an integral sequence measuring the polynomial identities satisfied by A. I will discuss a characterization in terms of Young diagrams in case such sequence is polynomially bounded and A is a non necessarily associative algebra.

Constructing generators of unit groups of group algebras of finite commutative groups

W.A. de Graaf

Let G be a finite abelian group, and $\mathbb{Z}G$ its integral group algebra. We describe an algorithm for constructing generators of the group of units $U(\mathbb{Z}G)$ of $\mathbb{Z}G$. The strategy is to split $\mathbb{Q}G$ as a sum of field extensions of $\mathbb{Q}$, then use algorithms for finding generators in unit groups of orders in number fields (corresponding to the Dirichlet unit theorem). Like this we find a group of which the group we are looking for is a finite index subgroup. We describe a method to find this subgroup. To illustrate the algorithm we will give generators of $U(\mathbb{Z}G)$ for several abelian groups G. We also indicate how this algorithm can be extended to yield generators of arithmetic groups corresponding to diagonalisable algebraic groups.

Fitting core and supersolvable groups

H. Heineken

A class A of groups will be exhibited with the following property: If G is a group of the class A and H a supersolvable group, then the normal product GH is supersolvable (equivalently: A belongs to the Fitting core of the class of supersolvable groups). Joint work with J.C.Beidleman

Groups and Set Theoretic Solutions of the Yang-Baxter equation

E. Jespers

In recent years there has been quite some interest in the "simplest" solutions of the Yang-Baxter equation. Such solutions are involutive bijective mappings $r : X \times X \to X \times X$, where X is a finite set, so that $r_1 r_2 r_1 = r_2 r_1 r_2$, with $r_1 = r \times id_X$ and $r_2 = id_X \times r$. In case r satisfies some non-degeneracy condition, Gateva-Ivanova and Van den Bergh, and also Etingof, Schedler and Soloviev, gave a beautiful group (monoid) theoretical interpretation of such solutions. Such groups (monoids) are said to be of I-type. In this lecture we give a survey of recent results on the algebraic structure of these groups (monoids) and their group (monoid) algebras.

On some infinite dimensional linear groups

L.A. Kurdachenko

Let F be a field and A a vector space over F. Denote by $GL(F, A)$ the group of all F-automorphisms of A. The subgroups of $GL(F, A)$ are called the linear groups. Linear groups have played a very important role in algebra and other branches of mathematics. If $\dim_F(A)$ (the dimension of A over F) is finite, n say, then a subgroup G of $GL(F, A)$ is a finite dimensional linear group. The theory of finite dimensional linear groups is one of the best developed in Group Theory. However, the study of the subgroups of $GL(F, A)$ in the case when A has infinite dimension over F has been much more limited and normally requires some additional restrictions. We consider here some approaches to the study of infinite dimensional linear groups. More concretely, we consider the linear groups with boundedly finite G-orbits of elements, the linear groups with boundedly finite dimensional G-orbits of elements, the linear groups with finite G-orbits of subspaces and also some generalization of finitary groups.

On some properties of nilpotent groups of class at most 2

D. Lenzi

We examine some interesting properties of nilpotent groups of class at most 2. Among the other things, we see that a sufficient condition for a group $(G, +)$ to be nilpotent of class at most 2 is that the following property (j') holds. In particular, if for any $x \in G$ there is a unique $y \in G$ such that $2y = x$, then property (j') is also sufficient.
(j') The operation $\rightarrow$ defined by putting, for any $x, z \in G$, $x \rightarrow z = z - x + z$, coincides with the analogous operation $\rightarrow'$ defined on G by a suitable commutative group operation $+'$.
Moreover we prove that a group $(G, +)$ of a prime exponent is nilpotent of class at most 2 if and only if the set χ^+ of the cosets associated with the cyclic sub-groups of G coincides with the set $\chi^{+'}$ of the cosets determined by a commutative group operation $+'$ on G.

Reversible and Duo Group Rings

Y. Li

Let R be an associative ring with identity. R is called reversible if $\alpha\beta = 0$ implies $\beta\alpha = 0$, and it is called symmetric if $\alpha\beta\gamma = 0$ implies $\alpha\gamma\beta = 0$ for all $\alpha, \beta, \gamma \in R$. The reversibility property, a natural generalization of commutativity, has been exploited by various authors over the years; but apparently the name was introduced by Cohn in 1999, who noted that the Köthe conjecture holds for the class of reversible rings. An associative ring R is called left (right) duo if every left (right) ideal is an ideal, and R is said to be duo if it is both left and right duo. In this talk, we present some recent results on reversible group rings, duo group rings, and graded reversible group rings; and we mention several open problems.

On decompositions of the longest element of S_n and the combinatorics of certain Kazhdan-Lusztig cells

C. Pallikaros

We study certain decompositions of the longest element of S_n and investigate their relation with increasing and decreasing subsequences occurring in the row form of some elements in S_n and the Kazhdan-Lusztig cells to which these elements belong. We discuss how this is related to the representation theory of the symmetric group.

On n-uniqueness of amalgams

E. Pastori

We discuss when certain systems of structures can be amalgamated together and when the result is unique in the light of the recent paper of Hrushovski “Groupoids, imaginaries and internal covers”. In particular, we analyze a specific combinatorial structure and we show how to translate the n-uniqueness problem in a group theoretic language in order to solve it.

On the (non) contractibility of the simplicial complex associated to the coset poset of a classical group

M. Patassini

Let G be a finite group. Let Δ be the simplicial complex associated to the coset poset of G. Let $\chi(\Delta)$ denote the Euler characteristic of Δ and let $\tilde{\chi}(\Delta) := \chi(\Delta) - 1$ be the reduced Euler characteristic. In [1] Brown noted that

$$P_G(-1) = -\tilde{\chi}(\Delta)$$

where $P_G(s)$ is the Dirichlet polynomial associated to G. It is a well-known fact that if Δ is contractible, then its reduced Euler characteristic $\tilde{\chi}(\Delta)$ is zero. In [1], Brown proved that if G is a finite soluble group, then $P_G(-1) \neq 0$. So, in this case, the simplicial complex Δ is not contractible. Moreover Brown conjectured that $P_G(-1) \neq 0$ for each finite group G. In our work, we prove the Brown conjecture for the classical groups which do not contain a graph automorphism, with some exceptions.

[1] K. S. Brown, *The coset poset and the probabilistic zeta function of a finite group*, J. Algebra 225 (2000), 989-1012.

Strong Forms of Residual Finiteness - Graphs of Groups and Generalized Baumslag-Solitar Groups - Quotients of Generalized Baumslag-Solitar Groups

D.J.S. Robinson

Lecture 1: Strong forms of residual finiteness.

We discuss strengthened forms of residual finiteness, including the property of being extended residually finite (ERF), i.e., each subgroup is closed in the profinite topology. We will review recent work which classifies the nilpotent groups, locally finite groups and FC-groups which are ERF.

Lecture 2: Graphs of groups and generalized Baumslag-Solitar groups.

After reviewing graphs of groups and their fundamental groups, we concentrate on the case where all the vertex groups are infinite cyclic, when the fundamental groups are called generalized Baumslag-Solitar (GBS-)groups. A survey of the properties of

GBS-groups is given. Then we consider their homology in low dimensions. A method for computing the Schur multiplier is described.

Lecture 3: Quotients of generalized Baumslag-Solitar groups.

We consider what can be said of various quotient groups of a GBS-group and define the concept of a geometric GBS-quotient. A classification is given of the GBS-free groups, i.e., GBS-groups which have no proper geometric quotients other than infinite cyclic groups.

On a generalization of the minimal non-nilpotent groups

F. Russo

I will discuss a joint work with N. Trabelsi in which it is extended the classic description of minimal non-nilpotent groups of

[1] M. F. Newman and J. Wiegold, Groups with many nilpotent subgroups, Arch. Math. 15 (1964), 241-250;

[2] L. A. Shemetkov, O. Yu. Schmidt and finite groups, Ukr. Math. J. (5) 23 (1971), 482-486;

[3] H. Smith, Groups with few non-nilpotent subgroups, Glasgow Math. J. 39 (1997), 141-151.

L_{10}-free groups

R. Schmidt

If L is a lattice, a group is called L-free if its subgroup lattice has no sublattice isomorphic to L. It is easy to see that L_{10}, the subgroup lattice of the dihedral group of order 8, is the largest lattice L such that every finite L-free p-group is modular. Therefore the finite L_{10}-free groups form an interesting class of groups lying between the modular groups and the groups with modular Sylow subgroups. We shall see that they are still near to the modular groups and we shall study more closely L_{10}-free $\{p, q\}$-groups (p and q primes).

Group rings and their group of units

S.K. Sehgal

I. Lie Properties in Group Rings

We begin with a survey of some classical results classifying *Lie nilpotent*, *Lie solvable* and *Lie n-Engel* group algebras FG.
Then we discuss the relationship between the Lie nilpotency class of FG and the nilpotency class of its unit group, $\mathcal{U}(FG)$. In this framework, we present results of Shalev and the activity that followed.
Lastly we study when the symmetric elements (the skew-symmetric elements), $(FG)^+$ ($(FG)^-$, respectively) of FG with respect to the *classical* involution of FG, namely that induced on FG from the map $g \mapsto g^{-1}$, $g \in G$, satisfy the above Lie properties. Generalization of these results to other kinds of involutions will be reviewed.

II. A Conjecture of Brian Hartley

Let $\mathcal{U}(FG)$ be the group of units of the group algebra FG. Around 1980 Hartley made the following

Conjecture *Assume that F is infinite and G is a torsion group. If $\mathcal{U}(FG)$ satisfies a group identity, then FG satisfies a polynomial identity.*

We shall speak of the solution to this conjecture and the extensive activity that ensued.

III. Generic Units

The question we discuss is:

When is an element $f(x) = \sum_{i=0}^{s} a_i x^i \in \mathbb{Z}C_n$, where $C_n = \langle x \rangle$, a unit for many values of n?

We shall present results as to when various type of units (Bass cyclic, bicyclic or generic) generate a free group.

On groups admitting a fixed-point-free elementary 2-group of automorphisms

C. Sica

Consider first locally finite groups G admitting a fixed-point-free four-group of automorphisms. A theorem of Bauman tells us that if G is finite, then G' is nilpotent. A simple inverse limit argument shows that in general G' is locally nilpotent. The first author showed in [2] that in fact the normal closure of any element of G' is hypercentral. Lie algebras admitting a fixed-point-free four-group of automorphisms have been studied in [3]. One of the results obtained there is that if a locally soluble Lie algebra L admits a fixed-point-free four-group of automorphisms, then the derived subalgebra $[L, L]$ is a sum of nilpotent ideals. The authors in [4] proved a similar result for locally finite groups:

Theorem 1. *Let G be a locally finite group admitting a fixed-point-free four-group of automorphisms. Then the derived group G' is a product of normal in G nilpotent subgroups.*

Consider now the action of an elementary abelian 2-group on a finite group G. First author in [1] proved the following:

Theorem 2. *There exists a function $f(x, y)$ of x and y with the following property. Let A be the elementary group of order 2^n acting fixed-point-freely on a finite group G that is solvable with derived length k. Then G has a normal series $1 = N_n \leq \cdots \leq N_1 \leq N_0 = G$ all of whose quotients are nilpotent and the class of N_{i-1}/N_i is at most $f(k, i)$ for $i = 1, \ldots, n$.*

The proof of the previous result given in 1988 was rather long and somewhat complicated; we present a new, much shorter, proof.

[1] P. Shumyatsky, *Groups with regular elementary 2-groups of automorphisms*, Algebra and Logic **27** (1988), 447457.

[2] P. Shumyatsky, *Locally finite groups with Carter subgroup which is with a fixedpoint-free four-group*, (Russian) Investigations of algebraic systems, Sverdlovsk, (1989), 131-133.

[3] P. Shumyatsky, *Groups and Lie algebras with a fixed-point-free four-group of automorphisms*, Comm. Algebra **24,** No.12, (1996), 3771-3785.

[4] P. Shumyatsky and C. Sica, *On groups admitting a fixed-point-free four-group of automorphisms* J. Group Theory, to appear.

Fixed-point-free elements in p-groups

P. Spiga

In this talk we prove that there exists no function $F(m, p)$ (where the first argument is an integer and the second a prime) such that, if G is a finite permutation p-group with m orbits, each of size at least $p^{F(m,p)}$, then G contains a fixed-point-free element. In particular, this gives an answer to a conjecture of Peter Cameron.

Minimal algebras with respect to their *-exponent

E. Spinelli

Let $(A, *)$ be a $*$-PI algebra with involution over a field of characteristic zero and let $c_m(A, *)$ denote its m-th $*$-codimension. Giambruno and Zaicev proved that, if A is finite dimensional, there exists the $\lim_{m\to+\infty} \sqrt[m]{c_m(A, *)}$, and it is a non-negative integer, which is called the $*$-*exponent* of A ([2]). As a consequence of the presence of this invariant, in a natural manner in [1] the definition of *minimal algebras with respect to their $*$-exponent* (or simply $*$-*minimal*) was introduced and the question of characterizing $*$-minimal algebras was studied. In particular, it was proved that for any finite dimensional $*$-minimal algebra with involution A there exists an m-tuple, $(A_1, \ldots, A_m)$, of finite dimensional $*$-simple algebras allowing to construct a block triangular matrix algebra with involution, $UT_*(A_1, \ldots, A_m)$, which is $*$-PI equivalent to A.

Motivated by a conjecture of [1], in the present talk we discuss (and solve) the "converse" problem, namely whether the algebra $UT_*(A_1, \ldots, A_m)$ is $*$-minimal for any choice of the algebras A_j. In this framework a fundamental role will be played by the representation theory of the symmetric group. This is a joint work with O.M. Di Vincenzo.

[1] Di Vincenzo, O.M., La Scala, R.: *Minimal algebras with respect to their $*$-exponent*, J. Algebra **317** (2007), 642-657.

[2] Giambruno, A., Zaicev, M.: *Involution codimension of finite dimensional algebras and exponential growth*, J. Algebra **222** (1999), 471-484.

Quasinormal subgroups of finite p-groups

S.E. Stonehewer

The problem of finding the (interesting) quasinormal subgroups of finite groups reduces easily to p-groups. Here the systematic search begins with many positive results and continues for some way. But then suddenly something very strange seems to occur ...

Totally inert simple groups

A. Tortora

Let G be a group. A subgroup H of G is called *inert* if $|H : H \cap H^g|$ is finite for all g in G. Obviously normal subgroups, finite subgroups and subgroups of finite index are inert. It is less obvious that permutable subgroups are inert (see D. J. S. Robinson, *On inert subgroups of a group*, Rend. Sem. Mat. Univ. Padova **115** (2006), 137–159.). If every subgroup of G is inert, then G is said to be *totally inert*, briefly TIN. It is easy to see that the class of FC-groups is properly contained in the class of TIN-groups. Soluble TIN-groups were studied in [3], whereas in [1] it was proved that are no locally finite simple TIN-groups. Since there exist locally (abelian-by-finite) simple groups which are not locally finite and with numerous infinite inert subgroups (see [2]), it is reasonable to wonder if locally graded simple groups can be TIN. Clearly, simple TIN-groups exist: the groups constructed by Olshanskii are simple and quasi-finite. In this talk we show some properties of simple TIN-groups and answer to the above question.

[1] V. V. Belyaev, M. Kuzucuoğlu and E. Seçkin, *Totally inert groups*, Rend. Sem. Mat. Univ. Padova **102** (1999), 151–156.

[2] M. R. Dixon, M. J. Evans and H. Smith, *Embedding groups in locally (soluble-by-finite) simple groups*, J. Group Theory **9** (2006), 383–395.

[3] D. J. S. Robinson, *On inert subgroups of a group*, Rend. Sem. Mat. Univ. Padova **115** (2006), 137–159.

Soluble minimal non-(finite-by-Baer)-groups

N. Trabelsi

If $\mathfrak{X}$ is a class of groups, a group G is said to be a minimal non-$\mathfrak{X}$-group if it is not an $\mathfrak{X}$-group but all of whose proper subgroups are $\mathfrak{X}$-groups. We will denote minimal non-$\mathfrak{X}$-groups by $MN\mathfrak{X}$-groups. In [1] a complete description of AN^*-groups, that is infinitely generated $MN\mathfrak{N}$-groups having a maximal subgroup, is given, where $\mathfrak{N}$ denotes the class of nilpotent groups. In [2] locally graded $MN\mathfrak{F}\mathfrak{A}$-groups are characterised, where $\mathfrak{F}$ and $\mathfrak{A}$ denotes respectively the class of finite and abelian groups. In [3], the class $\mathfrak{B}$ of Baer groups is considered and it is proved that if G is an infinite soluble $MN\mathfrak{B}$-group, then G/G'' is an AN^*-group and an example of an infinite soluble $MN\mathfrak{B}$-group which is not a $MN\mathcal{N}$-group is given. Here we generalise the result of Xu by proving that if G is a soluble $MN\mathcal{F}\mathcal{B}$-group (respectively, infinite soluble $MN\mathcal{B}$-group), then for all integer $n \geq 2$, $G/\gamma_n(G')$ is a $MN\mathfrak{F}\mathfrak{A}$-group (respectively, an AN^*-group).

[1] B. Bruno and R. E. Phillips, On minimal conditions related to Miller-Moreno type groups, *Rend. Sem. Mat. Univ. Padova* **69** (1983) 153-168.

[2] M. F. Newman and J. Wiegold, Groups with many nilpotent subgroups, *Arch. Math.***15** (1964) 241-250.

[3] M. Xu, Groups whose proper subgroups are Baer groups, *Acta. Math. Sinica* **40** (1996) 10-17.

Profinite and Residually Finite Groups

J.S. Wilson

Lecture 1 will be an ab inition introduction to profinite groups but it will arrive quickly at topics in which there has been recent progress. Lecture 2 will be concerned with branch groups (abstract and profinite), and examples and an internal characterization will be described. Lecture 3 will be concerned with the existence of well-placed free subgroups in finitely presented groups, and is partly motivated by elementary linear algebra.

Limits and laws with parameters of the Thompsons group F

R. Zarzycki

Let F be the (Thompson's) group $\langle x_0, x_1 | [x_0 x_1^{-1}, x_0^{-i} x_1 x_0^i], i = 1, 2\rangle$. Let $G_n = \langle y_1, \ldots, y_m, x_0, x_1 | [x_0 x_1^{-1}, x_0^{-1} x_1 x_0], [x_0 x_1^{-1}, x_0^{-2} x_1 x_0^2], y_j^{-1} g_{j,n}(x_0, x_1), 1 \leq j \leq m\rangle$, where $g_{j,n}(x_0, x_1) \in F$, $n \in \mathbb{N}$, be a family of groups isomorphic to F and marked by $m+2$ elements. If the sequence $(G_n)_{n<\omega}$ is convergent in the space of marked groups and G is the corresponding limit we say that G is an F-limit group. We prove that a non-trivial free product of groups cannot be an F-limit group in the space of groups marked by three elements. The same statement holds for HNN-extensions of F over finitely generated subgroups. On the other hand we find an infinitely generated $H < F$ such that the centralized HNN-extension of F over H occurs as an F-limit group. The results above are based on some theorems concerning laws with parameters in F. In particular we have found several constructions of such laws. On the other hand we formulate some very general conditions on words with parameters $w(y, a_1, \ldots, a_n)$ over F which guarantee that the inequality $w(y, \bar{a}) \neq 1$ has a solution in F. This generalizes a theorem of M. Abert that F does not satisfy any law without parameters.