ABSTRACTS

FINITE GROUPS WITH SUPERSOLUBLE NON-SUBNORMAL SUBGROUPS

Adolfo Ballester-Bolinches

A finite group in which every subgroup is normal is a Dedekind group and one in which every subgroup is subnormal is nilpotent. These conditions have been generalised in a number of ways. Our aim in this talk is to consider the structure of finite groups whose non-subnormal subgroups lie in some restricted classes of groups. All classes we consider are subclasses of the class of supersoluble groups.

GROUPS WITH ALL SUBGROUPS SUBNORMAL

Carlo Casolo

Although I believe that some aspects of it could be further improved, our knowledge of the class $\mathfrak{N}_1$ of groups in which every subgroup is subnormal, whose study strated in the sixties, is by now rather satisfactory. In this series of lectures I would like to present the principal results of an investigation which has benefitted from the contribution of many authors (Roseblade, Heineken and Mohamed, Hartley, Mhöres, Menegazzo, H. Smith, just to cite a few), and give a, necessarily limited, idea of some proofs.

In the first lecture, I will try to motivate the interest in $\mathfrak{N}_1$-groups by looking at them in the wider frame of the study of generalized nilpotency conditions, I will talk about the known methods for constructing non-nilpotent $\mathfrak{N}_1$-groups, and recall the basic theorem of Roseblade and its extensions.

In the second lecture I will firstly consider the problem of solubility of $\mathfrak{N}_1$-groups; then I intend to give a complete (and almost elementary) proof of the nilpotency of torsion-free $\mathfrak{N}_1$-groups.

In the third lecture I will illustrate some more specific results on the structure of non-nilpotent $\mathfrak{N}_1$-groups, which in a sense represent the actual state of the theory, and discuss what, in my opinion, has yet to be done.

CERTAIN RANK CONDITIONS ON GROUPS

Martyn R. Dixon

In 1948 Mal'cev defined the general rank and special rank of a group G. These definitions were based on an earlier definition of the rank of an abelian group made by Prüfer in 1924. A group G has *finite (or Prüfer or special) rank* r if every finitely generated subgroup of G can be generated by

r elements and r is the least integer with this property. If no such integer r exists then the group has infinite rank. The rank of the group G is denoted by $r(G)$. In this series of talks I shall give an outline of much of the theory and examples concerned with groups of finite rank and related rank conditions. Much of this theory was established by many famous mathematicians in the past, indicating the incredible importance of the subject, and I hope to indicate some of the history of this topic in my talks. I shall also indicate some recent results and will mention some well-known open questions.

PRODUCTS OF SUBGROUPS WITH ADDITIONAL PERMUTABILITY CONDITIONS

Hermann Heineken

The talk will concentrate on products AB with XB = BX and AY = YA for all subgroups X of A and Y of B (mutually permutable products), and on the case that A and B are nilpotent.

PRO-FINITE GROUPS AND LIE RINGS

David M. Riley

Pro-finite Lie rings are the Lie-ring-theoretic analogues of pro-finite groups. Lie-theoretic techniques have often been used to study groups and pro-finite groups in particular. For example, Zelmanov's celebrated proof of Platonov's conjecture that every periodic pro-p groups is locally finite is essentially an application of some of his deep results concerning Lie algebras. Many of the problems studied in pro-finite group theory have natural analogues for pro-finite Lie rings. In particular, Leland McInnes and I have been able to prove the precise analogue of Zelmanov's aforementioned theorem: every Engelian pro-finite Lie ring is locally nilpotent.

UNITS IN INTEGRAL GROUP RINGS

Sudarshan Sehgal

This is a survey talk about units in $\mathbb{Z}G$. We shall present torsion units, Bass cyclic units, Hoechsmann units, alternating units, generic units, bicyclic units, central units and free groups of units.

PRODUCTS OF GROUPS AND LOCAL NEARRINGS

Yaroslav Sysak

Groups which can be written as a product $G = AB$ of two of its subgroups A and B have been studied by many authors. In these investigations groups of the form $G = AB = AM = BM$ where M is a normal subgroup of

G play a particular role. If $A \cap B = A \cap M = B \cap M = 1$, there is an interesting connection between these groups and nearrings which are a generalization of associative rings in the sense that their additive groups is not necessarily abelian and only one distributive law holds. For instance, if R is a local nearring, i.e. a nearring with identity 1 in which the set L of all non-invertible elements forms a subgroup of the additive group of R, then the set $1 + L$ is a subgroup of the multiplicative group of R acting on L by a suitable multiplication such that the semidirect product $L \rtimes (1 + L)$ is a group G of the above form where the normal subgroup M is isomorphic to L and the subgroups A and B are isomorphic to $1 + L$.

In the lectures we will discuss in detail some aspects of this connection and, in addition, consider certain structural questions about such groups with cyclic-by-finite subgroups A and B. In particular, it will be shown that every group $G = AB$ with A and B having cyclic subgroups of index at most 2 is metacyclic-by-finite and that every local nearring with cyclic-by-finite multiplicative group must be finite.

FINITE GROUPS WITH MINIMAL 1-PIM

Thomas Weigel

For a finite group G and a prime number p the 1-PIM Φ_1^G is the projective indecomposable $\mathbb{F}_p[G]$-module with head and socle isomorphic to the trivial $\mathbb{F}_p[G]$-module $\mathbb{F}_p$. An easy argument shows that its $\mathbb{F}_p$-dimension is a multiple of the order of a Sylow p-subgroup P of G, i.e., $c_p(G) := \dim_{\mathbb{F}_p}(\Phi_1^G)/|P|$ is a positive integer. The finite group G is said to have minimal 1-PIM (at p) if $c_p(G) = 1$.

If G possesses a p'-Hall subgroup, then G has a minimal 1-PIM (at p). However, in general the converse is not true since $G = \mathrm{SL}_2(p)$ satisfies $c_p(G) = 1$ for all prime numbers p.

Recently, we - Gunter Malle and myself - obtained a classification of all finite simple groups G and prime numbers p satisfying $c_p(G) = 1$. From this classification one deduces easily that finite solvable groups are characterized by the property of having minimal 1-PIM's for all prime numbers p. Furthermore, it also shows that for small prime numbers $p \in \{2, 3, 5\}$ the property of having minimal 1-PIM is equivalent to the existence of a p'-Hall subgroup.